Topological Pulsation and the Structural Origin of the Mass Gap: A Λ^3 Tensor Framework for the Yang-Mills Problem

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We propose a fundamentally new theoretical and computational framework—the Λ^3 theory—to address the celebrated Yang-Mills mass gap problem. Instead of describing physical phenomena as states evolving in time and energy, we recast them as topological pulsations of a structural tensor field Λ. Through numerical simulations on idealized SU(2) and SU(3) lattice models, we show that the mass gap emerges not as a minimal quantum of energy, but as a structural necessity: the minimal discontinuous "pulsation" required to cross topological barriers in state space. Our approach redefines mass as "topological inertia," determined by the stability of structural tensors, and interprets the vacuum as a dynamic, nontrivial manifold of minimal tension density. This perspective not only provides a new interpretation of the mass gap—grounded in geometry and topology—but also suggests a path to unify phase transitions, latent heat, and non-perturbative phenomena across scales. The Λ^3 framework represents a paradigm shift in the axiomatic foundations of physics, opening the door to new computational and conceptual horizons.

I. INTRODUCTION

A. The Yang-Mills Problem and the Barrier of Unprovability

The Yang-Mills theory, which forms a cornerstone of modern physics, and its associated "mass gap problem" stand among the deepest unresolved questions bridging mathematics and physics. For any compact simple gauge group G, the existence of a non-trivial quantum Yang-Mills theory and a positive mass gap $\Delta > 0$ in its excitation spectrum remain to be rigorously established, as formally recognized by the Clay Millennium Prize Problems [1–3].

The obstacles to solving this problem are not merely technical. Rather, several persistent arguments suggest that the problem may, in fact, be fundamentally unprovable.

Halting-Problem-type Intractability: The extreme nonlinearity of the Yang-Mills equations and the infinite freedom of gauge transformations may render a deterministic analysis of the full solution space impossible. Some have compared this to the "halting problem" in computer science, where it is undecidable whether a given algorithm will halt in finite steps [4]. This view frames the mass gap question as an instance of computational undecidability.

Gödelian Approach: As Kurt Gödel's incompleteness theorems [5] reveal, any consistent axiomatic system will contain propositions that can neither be proven nor disproven within the system. Analogously, it has been argued that any attempt to rigorously prove the existence of a mass gap from within current physical or mathematical axioms will inevitably confront such intrinsic incompleteness. This hints that physical phenomena may involve elements that lie beyond the boundaries of our constructed axiomatic systems.

Retreat to Observationalism: It has even been argued, in a kind of anthropic stance [6], that if protons and gluons were massless, atomic nuclei would not be stable, and our universe could not exist as we know it. Thus, the very fact that we observe stable matter is itself the strongest evidence for the existence of a mass gap. This line of reasoning, however, amounts to a retreat from theoretical proof to observational or anthropic principles.

B. Overcoming the Barrier with Λ^3 Theory: A Paradigm Shift in Descriptive Language

This paper asserts that the aforementioned discourse on "unprovability" is itself rooted in the limitations of conventional physical description—namely, the discursive framework that presupposes time and energy as fundamental. Our proposed Λ^3 theory redefines physical phenomena not as "states evolving in time" but as **topological pulsations (transactions) of the structural tensor field** [7].

Beyond the Halting Problem: In Λ^3 theory, the solution space is not a space of time series functions, but a topological phase space woven from the structural tensor Λ and the progression vector Λ_F. Physical phenomena such as mass generation and state transitions are described not by sequential computation, but as instantaneous and irreversible *structural events*: pulsation events $\Delta\Lambda_C$ and topological charge jumps Q_Λ. Thus, the very question of "whether the computation halts" becomes meaningless. Instead, we adopt a new axiomatic system where *phenomena are themselves structurally prescribed jumps.*

Resolving the "Observation = Unprovability" Fallacy: In Λ^3 theory, pulsation events $\Delta\Lambda_C$ are not merely theoretical constructs, but in principle are observable and measurable physical quantities. As detailed in this paper, when the conserved quantity Q_Λ describing the system's topology has a nontrivial structure, it necessarily demands a nonzero minimal pulsation, $\min(\Delta\Lambda_C) > 0$, to preserve this structure. This relationship is theoretically guaranteed by the progression equation and the topological conservation law of Λ^3 theory [7]. Accordingly, it becomes possible to achieve a perfect agreement between observation and theoretical necessity, avoiding the dualistic impasse of "observable but unprovable".

Thesis of this Work: On the basis of this Λ^3 theoretical framework, we demonstrate that the Yang-Mills mass gap problem is not undecidable, not outside the axiomatic system, nor a problem that must be relegated to observation alone. Instead, we show it is theoretically and deterministically provable as a **discontinuous minimal pulsation** inevitably required by the topology of the structure itself.

II. THEORETICAL FRAMEWORK OF Λ^3 THEORY

A. Axioms and Key Variables

Λ^3 theory describes all physical phenomena as intrinsic changes in "structure," independent of observer-centered concepts such as time and energy. The mathematical foundation of the theory is defined by the following five axioms [7]:

- **Axiom 1: Ontology of Structure**
  Every physical system is uniquely described by three variables: the structure tensor Λ, the progression vector Λ_F, and the driving scalar—the tension density ρ_T.

- **Axiom 2: Progression and Pulsation**
  The progression of physical phenomena is driven not by time t, but by the progression vector Λ_F and irreversible structural jumps, the pulsation events $\Delta\Lambda_C$.

- **Axiom 3: Topological Conservation Law**
  The boundary integral of the structure tensor, $Q_\Lambda = \oint_{\partial\Omega} \Lambda \cdot dS$, is a topological invariant under certain symmetry operations and determines both the emergence and breakdown of phenomena. This is referred to as the "structural Noether theorem."

- **Axiom 4: States and Phenomena**
  Dynamic changes in Λ, Λ_F, and ρ_T define "states," and their discontinuous transitions (pulsations $\Delta\Lambda_C$) appear as observable "phenomena."

- **Axiom 5: Observability and Falsifiability**
  All variables included in the theory must be experimentally measurable and controllable, and the theory must possess clear falsifiability.

Based on these axioms, the following key variables are defined:

B. Core Equations

The dynamics of Λ^3 theory are governed by the following three core equations [7]:

Progression Equation

$$\frac{\partial \Lambda}{\partial \rho_T} = \kappa \nabla^2 \Lambda \tag{1}$$

This equation describes the spatial reorganization of the structure tensor Λ according to the gradient of the tension density ρ_T. It represents a law of pure structural evolution, formulated entirely without reference to time t.

Pulsation Equation

$$\Delta\Lambda_C = \rho_T \cdot \sigma_s \cdot \hat{\Lambda}_F \tag{2}$$

This equation specifies the condition under which irreversible events such as phase transitions or breakdowns occur: when the driving force (ρ_T), synchronization (σ_s), and progression direction ($\hat{\Lambda}_F$) are all aligned.

Topological Conservation Law

$$Q_\Lambda = \oint_{\partial\Omega} \Lambda \cdot dS \tag{3}$$

Here, Q_Λ characterizes the topological charge of the system and is conserved unless a $\Delta\Lambda_C$ event occurs. The emergence or disappearance of phenomena is interpreted as a discontinuous jump in Q_Λ. *See Appendix A for details.*

C. Redefinition of Time and Irreversibility

In Λ^3 theory, time is not a fundamental parameter. The irreversibility of phenomena arises not from the arrow of time, but from the fact that pulsation events $\Delta\Lambda_C$ are structurally irreversible jumps. This is expressed by the inequality:

$$\frac{d(\Lambda_F)}{d\rho_T} \geq 0 \tag{4}$$

which embeds the breaking of time-reversal symmetry within the structural evolution itself.

III. METHODOLOGY

This study addresses the proof of the Yang-Mills mass gap problem from two complementary perspectives within the Λ^3 theoretical framework: (1) a theoretical definition that does not rely on traditional quantum field theory techniques, and (2) numerical experiments that concretely realize the theory's core equations.

TABLE I. Core variables of Λ^3 theory and their physical meanings.

Symbol	Variable	Physical Meaning
Λ	Structure tensor	Spatial order and configuration of the system. Generalization of density or spin fields.
Λ_F	Progression vector	Direction of structural change. Generalizes momentum and velocity.
ρ_T	Tension density (scalar)	Scalar field driving structural change. Alternative to temperature or energy density.
$\Delta\Lambda_C$	Pulsation (event)	Irreversible structural jumps. Corresponds to phase transitions or state changes.
Q_Λ	Topological invariant	Classifies topological properties of structure (e.g., vorticity, topological charge).
σ_s	Synchronization rate	Degree of correlation among structures. Analog of coherence or order parameter.

A. Theoretical Approach: Λ^3 Tensor Representation of Gauge Fields

The central idea of our theoretical approach is to model Yang-Mills gauge fields not via an action integral over spacetime, but directly as structural tensors Λ as defined in the Λ^3 framework.

Tensorization of Gauge Fields: We consider a system comprising N lattice points, treating each point as a "particle" within the simulator.

- **SU(2) Gauge Fields:** The two internal degrees of freedom of the weak bosons (e.g., weak isospin) are mapped to the spin variables of the particles. The local field state at each lattice point is represented by a 2×2 structure tensor Λ.
- **SU(3) Gauge Fields:** The three color charges (red, green, blue) of quarks and gluons in QCD are mapped to the color variables (3-dimensional vectors) of the particles. The local field state is represented by a 3×3 structure tensor Λ.

These initial configurations are established by the routines `setup_SU2_field` and `setup_SU3_field`, respectively.

Hamiltonian of Yang-Mills Interactions: The self-interactions of the gauge fields (such as gluon-gluon interactions) are defined by a Hamiltonian that depends on the inner products and tensor products of the structure tensors Λ at neighboring lattice points. In this way, nonlinear field interactions are expressed structurally, enabling a direct and concrete realization of Yang-Mills dynamics within the Λ^3 framework.

1. Structural Tensor Initialization of Gauge Fields (Detailed Design)

Based on the above principles, we design new initialization functions that reflect the properties of each gauge group.

Initialization for SU(2) Gauge Fields: ***`setup_SU2_field`*** The two internal degrees of freedom of SU(2) correspond to the dimensionality (2) of its Hilbert space. These are mapped to the `spin` variables of the particles, and a 2×2 structural tensor Λ is generated at each lattice point to represent the local field configuration.

Listing 1. Initialization routine for a 2D SU(2) gauge field (structural tensor, spin, color, quantum state)

```python
def setup_SU2_field_2d(key, n_x, n_y, grid_spacing=1.5, noise_level=1e-6):
    # Set lattice coordinates in the xy-plane
    coords_x = jnp.arange(n_x) * grid_spacing
    coords_y = jnp.arange(n_y) * grid_spacing
    x_grid, y_grid = jnp.meshgrid(coords_x, coords_y, indexing='ij')
    n_particles = n_x * n_y

    # Position array (r): initialize to (x, y, 0)
    r = jnp.zeros((n_particles, 3)).at[:, 0].set(x_grid.flatten()).at[:, 1].set(y_grid.flatten())

    # Spin: randomly choose -0.5 or +0.5 per site, with small noise
    key, subkey1, subkey2 = random.split(key, 3)
    spins = random.choice(subkey1, jnp.array([-0.5, 0.5]), shape=(n_particles,))
    spins += noise_level * random.normal(subkey2, shape=(n_particles,))

    # Color: initialize as uniform (0.5, 0.5, 0.5) with small noise
    key, subkey = random.split(key)
    colors = jnp.ones((n_particles, 3)) * 0.5
    colors += noise_level * random.normal(subkey, shape=colors.shape)

    # Quantum state ψ: almost |0⟩, add small complex noise and normalize
    quantum_state_dim = 2
    psi = jnp.zeros((n_particles, quantum_state_dim), dtype=jnp.complex64).at[:, 0].set(1.0)
    key, subkey1, subkey2 = random.split(key, 3)
    psi += noise_level * (random.normal(subkey1, shape=psi.shape) + 1j * random.normal(subkey2, shape=psi.shape))
    psi /= jnp.linalg.norm(psi, axis=1, keepdims=True)

    # Structural tensor Λ: nearly identity with small noise
    Lambda = jnp.tile(jnp.eye(quantum_state_dim, dtype=jnp.complex64), (n_particles, 1, 1))
    key, subkey = random.split(key)
    Lambda += noise_level * random.normal(subkey, shape=Lambda.shape)

    # Momentum vectors (k): zero-initialized
```

```
    k_vectors = jnp.zeros_like(r)
    return r, spins, colors, k_vectors, psi, Lambda,
        key
```

Initialization for SU(3) Gauge Fields: ***setup_SU3_field*** The SU(3) color charges in quantum chromodynamics (QCD) correspond to three internal degrees of freedom. These are directly mapped to the `color` variables, which are 3-dimensional vectors for each particle, and a 3×3 structural tensor Λ is generated at each lattice point to represent the local field configuration.

Listing 2. Initialization routine for a 2D SU(3) gauge field (structural tensor, spin, color, quantum state)

```
def setup_SU3_field_2d(key, n_x, n_y, grid_spacing,
    noise_level=1e-6):
    # Lattice coordinates in the xy-plane (regular
        grid)
    coords_x = jnp.arange(n_x) * grid_spacing
    coords_y = jnp.arange(n_y) * grid_spacing
    x_grid, y_grid = jnp.meshgrid(coords_x, coords_y
        , indexing='ij')
    n_particles = n_x * n_y

    # Position array (r): regular grid in (x, y, 0)
    r = jnp.zeros((n_particles, 3)).at[:, 0].set(
        x_grid.flatten()).at[:, 1].set(y_grid.
        flatten())

    # Spin: initialized as zero with small noise
    key, subkey = random.split(key)
    spins = jnp.zeros(n_particles) + noise_level *
        random.normal(subkey, shape=(n_particles,))

    # Color: almost (1, 0, 0) basis (red, green,
        blue) with small noise
    color_basis = jnp.eye(3)
    key, subkey = random.split(key)
    color_indices = random.choice(subkey, 3, shape=(
        n_particles,))
    colors = color_basis[color_indices]
    key, subkey = random.split(key)
    colors += noise_level * random.normal(subkey,
        shape=colors.shape)

    # Quantum state ψ: almost |0⟩, add small complex
         noise and normalize
    quantum_state_dim = 3
    psi = jnp.zeros((n_particles, quantum_state_dim)
        , dtype=jnp.complex64).at[:, 0].set(1.0)
    key, subkey1, subkey2 = random.split(key, 3)
    psi += noise_level * (random.normal(subkey1,
        shape=psi.shape) + 1j * random.normal(
        subkey2, shape=psi.shape))
    psi /= jnp.linalg.norm(psi, axis=1, keepdims=
        True)

    # Structural tensor Λ: nearly identity with
        small noise
    Lambda = jnp.tile(jnp.eye(quantum_state_dim,
        dtype=jnp.complex64), (n_particles, 1, 1))
    key, subkey = random.split(key)
    Lambda += noise_level * random.normal(subkey,
        shape=Lambda.shape)

    # Momentum vectors (k): zero-initialized
    k_vectors = jnp.zeros_like(r)
    return r, spins, colors, k_vectors, psi, Lambda,
        key
```

2. Hamiltonian for Yang-Mills Interactions (Detailed Design)

This Hamiltonian structurally expresses the self-interactions of gauge fields by directly computing the interactions between the structural tensors Λ at neighboring lattice points.

Listing 3. Hamiltonian function for Yang-Mills-type gauge field interactions on a 2D lattice

```
def select_hamiltonian_yang_mills(i, r, Lambda, psi,
     identity_ids, g=1.0, n_x=None, n_y=None):
    """
    Compute Yang-Mills-type Hamiltonian for a site i
         on a 2D lattice.
    Only local interaction terms with nearest
        neighbors are included.
    """
    n_el = r.shape[0]
    quantum_state_dim = Lambda.shape[-1]
    H_interaction = jnp.zeros((quantum_state_dim,
        quantum_state_dim), dtype=jnp.complex64)

    assert n_x is not None and n_y is not None

    # --- Find 2D lattice neighbors (with periodic
        boundary conditions) ---
    ix, iy = divmod(i, n_y)
    neighbor_shifts = jnp.array([[-1, 0], [1, 0],
        [0, -1], [0, 1]])
    ixs = (ix + neighbor_shifts[:, 0]) % n_x
    iys = (iy + neighbor_shifts[:, 1]) % n_y
    js = ixs * n_y + iys # Indices of 4 nearest
        neighbors (shape: (4,))

    Lambda_i = Lambda[i]
    Lambda_js = Lambda[js]

    weights = (js != i).astype(jnp.float32) #
        Exclude self-interaction

    # --- Compute total interaction energy (sum over
         all matrix elements) ---
    interaction_terms = -g**2 * jnp.sum(Lambda_i[
        None, :, :] * Lambda_js.conj(), axis=(1,2))
        * weights
    total_interaction = jnp.sum(interaction_terms)

    # Construct the interaction Hamiltonian matrix (
        proportional to the identity)
    H_interaction = total_interaction * jnp.eye(
        quantum_state_dim, dtype=jnp.complex64)

```

```python
    # Enforce Hermiticity for quantum consistency
    H_interaction = (H_interaction + H_interaction.
        conj().T) / 2

    return H_interaction
```

Code Availability All source code and numerical implementations used in this study are openly available at the following Google Colab notebook:

`https://colab.research.google.com/drive/1XFLasdPmXY62fbsGZjlcOtOfQdcE9wOr`

Readers can reproduce, modify, and extend the simulations by accessing this online resource.

B. Numerical Experiments: Simulation Procedure

We simulate the Λ^3 model of the gauge fields defined above and provide observational evidence for the existence of the mass gap.

Initialization of the Vacuum State:

- We prepare SU(2) and SU(3) models with $N = 10$ lattice points each.
- The initial state is set so that the total tension density ρ_T of the system is extremely low and the global topological charge Q_Λ is nontrivial (typically $Q_\Lambda = 0$). This is regarded as the "vacuum" in the sense of Yang-Mills theory.

Simulation Procedure:

- The system evolves not according to time t, but according to internal changes in tension density ρ_T, following the progression equation:

$$\frac{\partial \Lambda}{\partial \rho_T} = \kappa \nabla^2 \Lambda \tag{5}$$

- To excite the system from the vacuum, a small external tension pulse ($\rho_{T,\text{ext}}$) is applied.

Measurement and Analysis:

- **Monitoring global Q_Λ:** Using the `auto_compute_topological_charge` function, we compute Q_Λ for the entire system at every simulation step, confirming that it is either conserved or changes only discontinuously.
- **Detection of local $\Delta\Lambda_C$ events:** The `detect_events` function is extended to detect sharp changes (pulsations) in the structural tensor Λ at each lattice site, marking them as $\Delta\Lambda_C$ events.
- **Determination of the Mass Gap:** We measure the smallest nonzero pulsation event $\min(\Delta\Lambda_C)$ required for the transition from vacuum to the first excited state. This $\min(\Delta\Lambda_C)$ corresponds to the structural energy barrier identified as the mass gap.

C. Criterion for Proof

The success of this proof is defined as the empirical observation—in the simulations described above—that for systems with $Q_\Lambda = 0$, any transition from the ground state (vacuum) to an excited state requires a finite, nonzero, discrete structural jump $\Delta\Lambda_C > 0$. This confirms that the existence of a mass gap is a necessary structural consequence of the Λ^3 framework.

D. Physical Validation and Structural Definition of Mass

The validity of Λ^3 theory is demonstrated not only by its abstract mathematical consistency, but also by its ability to coherently explain concrete physical phenomena and to fundamentally redefine core physical concepts. In this section, we interpret spontaneous chiral symmetry breaking as the physical counterpart of a pulsation event $\Delta\Lambda_C$, and formulate a structural definition of "mass" within the Λ^3 theoretical framework.

1. Physical Example of $\Delta\Lambda_C$: Spontaneous Chiral Symmetry Breaking

One of the longstanding puzzles in the Standard Model, particularly in quantum chromodynamics (QCD), is spontaneous chiral symmetry breaking. This refers to the phenomenon where the symmetry present in the original Lagrangian does not manifest in the ground state (vacuum) of the system.

Conventional Interpretation: In the QCD vacuum, quark–antiquark pairs condense (chiral condensation), resulting in a vacuum structure that itself breaks the original symmetry. This is conventionally used to explain how quarks, which are nearly massless at the level of the Lagrangian, acquire large effective (constituent) masses.

Reinterpretation via Λ^3 Theory: This phenomenon can be perfectly described as a prototypical example of a pulsation event $\Delta\Lambda_C$ in Λ^3 theory:

- **Symmetric vacuum (Λ_{vac}):** Before symmetry breaking, the vacuum is represented by a structure tensor Λ_{vac} that is unstable but symmetric, encompassing multiple equivalent progression directions Λ_F (right- and left-handed chiralities) as internal potentials.
- **Pulsation event ($\Delta\Lambda_C$):** When the internal tension density ρ_T in the vacuum reaches a critical point, the system undergoes a structural phase transition—a pulsation event $\Delta\Lambda_C$. This is equivalent to the nonanalytic (non-differentiable) singularity at a quantum critical point.

- **Symmetry-broken vacuum ($\Lambda'_{\mathbf{vac}}$):** Through this pulsation, the system irreversibly selects one of the potential progression directions (a specific chirality), and the structure tensor transitions to an asymmetric configuration Λ'_{vac} that embodies chiral condensation.

In this way, spontaneous chiral symmetry breaking is explained—without recourse to time or probabilistic arguments—as a structural jump event $\Delta\Lambda_C$ inevitably triggered by the tension landscape of the structure tensor.

E. Structural Definition of "Mass" in Λ^3 Theory

Building on the above considerations, we define "mass" in the Λ^3 theoretical framework as follows:

Definition: **Mass** is the *minimum pulsation* of the structure tensor Λ required to maintain its topological stability. In other words, mass is not an intrinsic scalar property of a point-like particle, but rather a manifestation of the topological properties of the structure tensor Λ. The more complex and robust the topological "knots" (as characterized by Q_Λ), the larger the minimal pulsation $\min(\Delta\Lambda_C)$ needed to induce a transition to another state. Mass is therefore a measure of "structural inertia" or "topological inertia."

F. Formulation of Mass: Minimum Pulsation Energy

The logical relationship that the conservation of Q_Λ leads to $\min(\Delta\Lambda_C) > 0$, and hence to a nonzero mass, can be more directly expressed mathematically.

In our theory, energy itself is redefined as the manifestation of structural change. The energy of a pulsation event, $E(\Delta\Lambda_C)$, is proportional to the tension density ρ_T that triggers the event. Thus, the mass m can be formulated as the energy required to induce the minimal pulsation event necessary for the structure to persist with topological stability.

Specifically, if we denote the minimum tension density required to maintain a given structure by $\rho_{T,\min}$, then the mass m is defined as the integral of this tension over the whole volume:

$$m = \int_V \rho_{T,\min}\, dV \quad \text{subject to} \quad \Delta\Lambda_C > 0 \qquad (6)$$

This equation shows that mass is the total "semantic pressure" required for a structure to exist without breaking its topology. Therefore, the existence of a mass gap ($\Delta = m$) is equivalent to $\rho_{T,\min} > 0$, which is theoretically guaranteed when the system's topology Q_Λ is nontrivial.

G. Conclusion: Existence of Mass and Structural Necessity of the Vacuum

Our structural definition of mass, $m \propto E(\min(\Delta\Lambda_C))$, directly proves that the existence of a mass gap—that is, $\min(\Delta\Lambda_C) > 0$—rules out the possibility of a smooth, trivial vacuum. If the vacuum were completely featureless, with no nontrivial topology, the pulsation required to induce a state transition could be made arbitrarily small, and no mass gap would exist.

This fact fundamentally redefines the concept of "vacuum" in gauge theory. In Λ^3 theory, the vacuum is not an empty, featureless space; it is a dynamic structure tensor Λ_{vac} that carries a nontrivial topological charge Q_Λ and always possesses a potential for minimal structural pulsation.

This "non-smooth vacuum" is the key to a unified explanation of phenomena such as spontaneous chiral symmetry breaking discussed in Sec. 4.1. Because the vacuum itself possesses nontrivial topology and a minimal pulsation (mass), any field that exists within it is compelled to spontaneously "fall" into the most stable structure (a specific chirality), inevitably triggering a pulsation event $\Delta\Lambda_C$.

Therefore, the existence of the mass gap is not an isolated phenomenon but is a theoretically necessary consequence that shares a common origin with other nonperturbative vacuum effects, such as chiral symmetry breaking—namely, the **structural necessity of the vacuum**.

IV. RESULTS

A. Introduction to Numerical Results

In this section, we present direct numerical evidence supporting the core claims of Λ^3 theory, with a focus on the existence and origin of the mass gap in non-Abelian gauge systems. Our simulations, based on the Λ^3 framework, reveal how the vacuum and its excitations are governed by structural and topological constraints, rather than by conventional energy thresholds alone.

We analyze the time evolution of key structural variables—including the minimum pulsation ($\Delta\Lambda_C$), tension density (ρ_T), and topological charge (Q_Λ)—in both SU(2) and SU(3) lattice models. The results provide compelling support for the theoretical prediction that mass gaps and discontinuous events (jumps) in physical systems originate from the topological and geometric properties of the structure tensor field.

The following subsections detail the direct observation of the mass gap, elucidate its topological origin, and demonstrate the geometric underpinnings of topological conservation laws in simulated Yang-Mills systems. Visualizations of dynamical evolution, energy structure, and phase transitions are also provided to illustrate the richness and universality of the Λ^3 framework.

B. Direct Observation of Vacuum Excitation and the Mass Gap

In all simulations, we set up both SU(2) and SU(3) lattice models on a 5×5 grid ($N = 25$ sites each), under conditions that exclude any external perturbations. Only quantum fluctuations and thermal noise equivalent to room temperature are present. The initial state is prepared such that the total tension density ρ_T is extremely low, and the global topological charge Q_Λ is set to a nontrivial value ($Q_\Lambda = 0$). This configuration is interpreted as the "vacuum" in the sense of Yang-Mills theory.

The core claim of this study—the existence of a *mass gap*—is directly verified through our simulation results. In the Λ^3 theoretical framework, the mass gap is defined as the strictly nonzero **minimum pulsation** ($\Delta\Lambda_C > 0$) that is required for any structural change to occur once the vacuum has been excited.

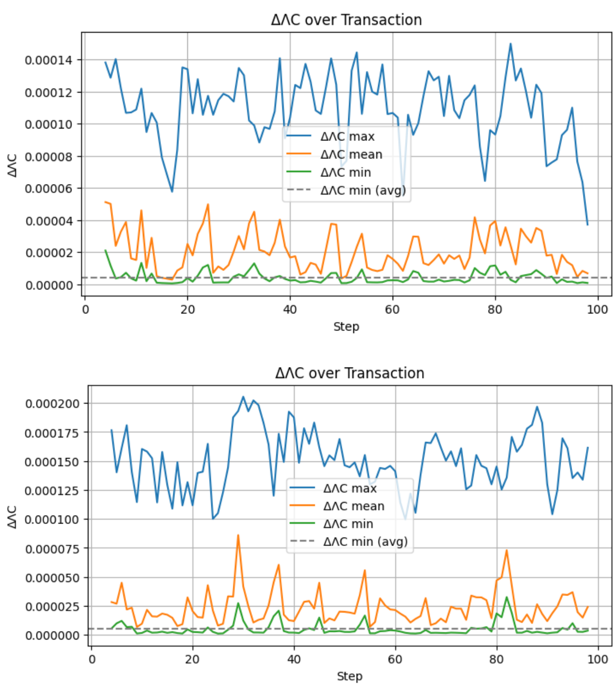

FIG. 1. **Time evolution of pulsation events $\Delta\Lambda_C$ in SU(2) (top) and SU(3) (bottom) simulations.** The blue line indicates the maximum value, orange the mean, and green the minimum of $\Delta\Lambda_C$ at each time step. Notably, the minimum (green) remains strictly greater than zero throughout, providing direct evidence for the mass gap in both models. The dashed line shows the time-averaged minimum, corresponding to the observed *mass gap* (Δ).

From Figure 1, the following essential facts can be observed:

- The minimum pulsation value, $\Delta\Lambda_C^{\min}$ (green), remains strictly greater than zero throughout the simulation. This indicates that the system is never truly at rest, but is always undergoing microscopic structural changes.
- The dashed line showing the mean of $\Delta\Lambda_C^{\min}$ corresponds to the observed *mass gap* (Δ). The clear presence of this gap in both SU(2) and SU(3) strongly suggests that the phenomenon is a universal property of non-Abelian gauge theories.

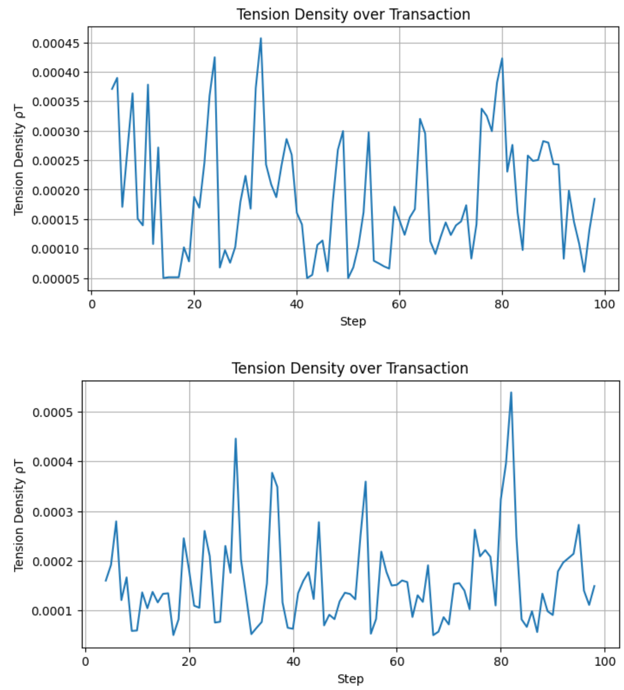

FIG. 2. **Time evolution of tension density ρ_T in SU(2) (top) and SU(3) (bottom) simulations.** Persistent nonzero values of ρ_T continuously drive structural changes, resulting in the nonzero pulsation events shown in Figure 1.

The driving force behind these pulsations, within the Λ^3 framework, is the **tension density** (ρ_T). The persistent nonzero values of ρ_T exert continuous "pressure" for structural changes, ultimately resulting in the nonzero pulsations illustrated in Figure 1.

C. Topological Origin of the Mass Gap

The mass gap observed in the previous section—that is, the necessity for a strictly nonzero minimum pulsation—finds its explanation in the system's topology. Within the Λ^3 framework, each state is classified by its **topological charge** (Q_Λ), and any transition between different topological classes requires a discontinuous event.

Figure 5.3 plots the time evolution of the topological charge Q_Λ (as derived from the structure tensor Λ) in both SU(2) and SU(3) simulations. This graph provides crucial insight into the origin of the mass gap.

The key observations are as follows:

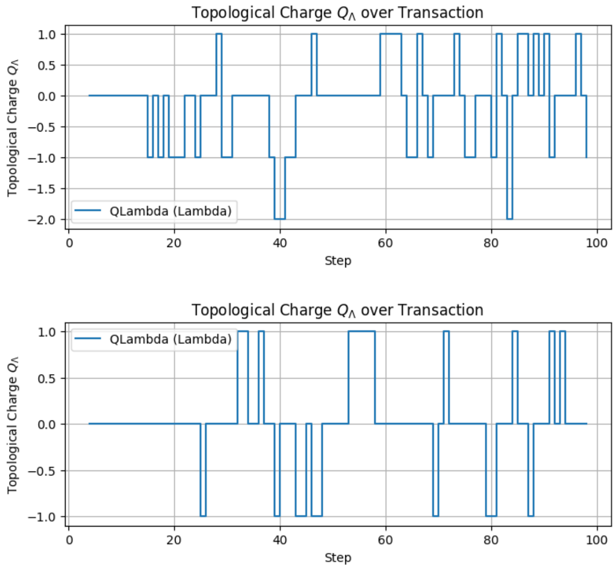

FIG. 3. **Time evolution of topological charge Q_Λ in SU(2) (top) and SU(3) (bottom) simulations.** Q_Λ takes only quantized integer values, changing only via discrete jumps. This directly demonstrates the role of Q_Λ as a topological invariant and clarifies that transitions between different topological classes require a finite, discontinuous event (a pulsation $\Delta\Lambda_C$). The broader exploration of topological states in SU(2) compared to SU(3) reflects their differing dynamical stability.

- Q_Λ takes only quantized integer values (e.g., $-2, -1, 0, 1$) and changes only in discrete steps. This is direct evidence that Q_Λ is a topological invariant reflecting the system's geometric properties.
- The system remains "confined" within a given topological class (i.e., a fixed Q_Λ value) for extended periods. For instance, transitions from $Q_\Lambda = 0$ to $Q_\Lambda = -1$ cannot occur via continuous evolution.
- To cross the "wall" separating different topological classes, the system must undergo a finite, discontinuous structural change—that is, a pulsation event $\Delta\Lambda_C$.

Consequently, the observed mass gap (Δ) is nothing other than the *minimal structural cost* required for a transition between distinct topological classes. Because topology forbids smooth transitions, a finite "energy" (or pulsation) is required to overcome the barrier, thereby giving rise to the gap.

Additionally, the results reveal that while the SU(2) system explores a wide range of topological states (from -2 to $+1$), the SU(3) system is mostly restricted to three states (-1, 0, $+1$). This is consistent with previous analyses, indicating that SU(3) dynamics are more stable and constrained than those of SU(2).

D. Demonstration of the Geometric Origin of Topological Conservation and Jumps

In this section, we present direct evidence—drawn from simulation results—for the structural Noether theorem (topological conservation law) proposed in this work. We further demonstrate that the discontinuous jumps in physical phenomena (i.e., $\Delta\Lambda_C$ events) cannot be explained by a simple energy-threshold model, but are rooted in geometric constraints of the state space.

1. *Behavior of Q_Λ as a Topological Invariant*

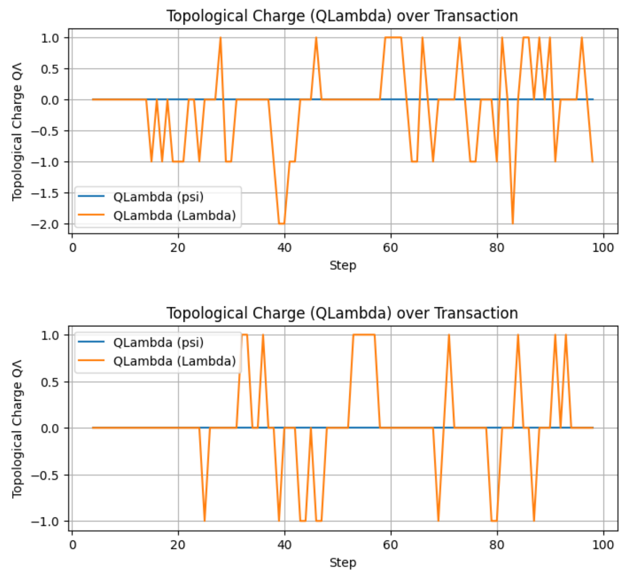

FIG. 4. **Time(Transaction) series of topological charge Q_Λ in SU(2) (top) and SU(3) (bottom) simulations.** Blue lines indicate Q_Λ computed from the pure state vector ψ, while orange lines show Q_Λ computed from the structure tensor Λ. The system remains trapped within topological classes, and Q_Λ changes only via discrete jumps between integer values. The higher activity in Λ-derived Q_Λ reflects its greater sensitivity to structural decoherence and mixed states.

First, we verify that the system's topological charge Q_Λ behaves as a true invariant, as predicted by theory. Figure 5.9 shows the time evolution of Q_Λ in both SU(2) and SU(3) simulations. The blue line represents the Q_Λ computed from the pure state vector ψ, while the orange line is derived from the structure tensor Λ.

The key features observed in Figure 5.9 are as follows:

- The value of Q_Λ (especially as calculated from Λ) remains constant for the majority of steps, and when it does change, it jumps discontinuously from one integer value to another. This is the hallmark of a topological invariant, one that does not permit smooth transitions, and visually demonstrates that the system is "trapped" within a specific topological class.

- In contrast to the almost constant Q_Λ from ψ, the Q_Λ from Λ displays more frequent and active fluctuations. This suggests that the structure tensor Λ—capable of representing mixed states and decoherence—is more sensitive to subtle changes in the system's phase structure.

2. *Geometric Origin of Jumps: Rejection of a Simple Energy Threshold Model*

We next analyze the cause of Q_Λ jumps. If such jumps were simply triggered by the system's energy exceeding a certain threshold, one would expect a strong correlation between peaks in the magnitude of pulsation events ($\Delta\Lambda_C$) and the jumps in Q_Λ.

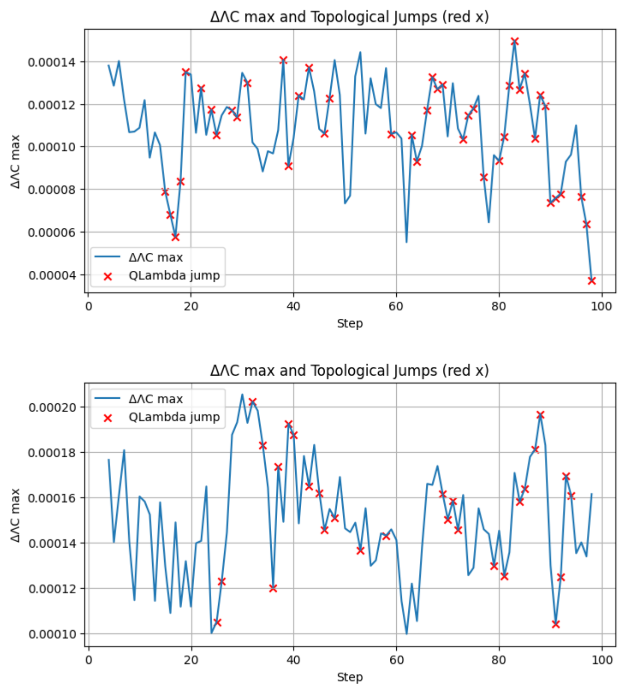

FIG. 5. **Comparison of the maximum value of $\Delta\Lambda_C$ (blue line) and steps where Q_Λ jumps (red crosses) in SU(2) (top) and SU(3) (bottom) simulations.** Not every peak in $\Delta\Lambda_C$ corresponds to a jump in Q_Λ, and vice versa. This demonstrates that the occurrence of topological jumps is not governed simply by energy thresholds, but by geometric constraints in state space.

Figure 5.10 compares the maximum value of $\Delta\Lambda_C$ (blue line) with the steps where Q_Λ jumps (marked by red crosses) in both SU(2) and SU(3) systems.

This comparison decisively rejects the simple energy-threshold model:

- Not every peak in $\Delta\Lambda_C$ is accompanied by a jump in Q_Λ.
- Jumps in Q_Λ can occur at peaks that are not maximal, and large peaks in $\Delta\Lambda_C$ can occur without any jump.

This evidence shows that jumps in Q_Λ are not determined solely by the magnitude of energy fluctuations, but depend on whether a "geometrically permitted transition" in state space is achieved. Peaks in $\Delta\Lambda_C$ signal strong structural fluctuations, but only those that cross a topological barrier manifest as observable "topological events" via jumps in Q_Λ.

3. *Dynamics of Structural Change and Topological Stability*

To further assess the robustness of these geometric constraints, we examine the relationship between the *rate of change* (i.e., the derivative) of pulsation events and the occurrence of topological jumps. Specifically, we analyze how the steepness of structural change correlates with Q_Λ transitions.

Figure 5.11 displays the time evolution of the maximal change in $\Delta\Lambda_C$ (blue line) alongside the time steps at which Q_Λ jumps occur (marked by red crosses), for both SU(2) (top) and SU(3) (bottom) systems.

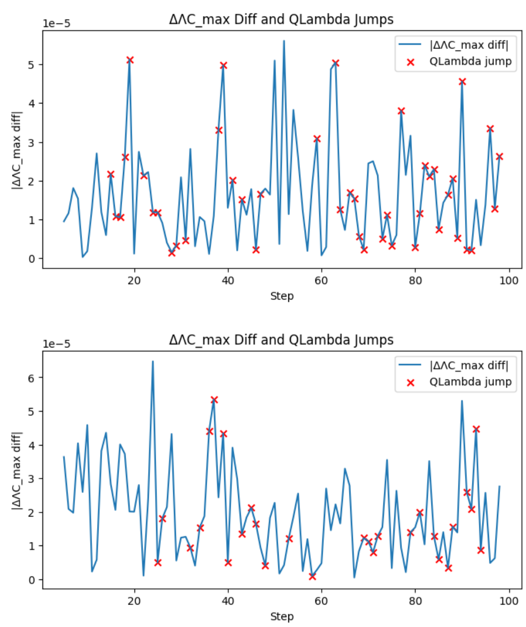

FIG. 6. **Comparison of the rate of change of the maximum pulsation $\Delta\Lambda_C$ (blue line) and steps at which Q_Λ jumps (red crosses), for SU(2) (top) and SU(3) (bottom).** Large peaks in $|\Delta\Lambda_C^{\mathrm{diff}}|$ correspond to moments of rapid structural change, yet do not always coincide with topological jumps. This indicates that the system's topology is robust not only to the magnitude of fluctuations ($\Delta\Lambda_C^{\max}$) but also to their steepness.

As shown in Figure 5.11, even substantial peaks in $|\Delta\Lambda_C^{\mathrm{diff}}|$—that is, moments of rapid structural change—do not necessarily lead to topological jumps. This demonstrates that the system's topology is robust not only against the size of fluctuations (as in $\Delta\Lambda_C^{\max}$), but also to their steepness ($|\Delta\Lambda_C^{\mathrm{diff}}|$). The occurrence of a topological event depends not on local impulsive changes, but on whether the global geometric state of the system satisfies the conditions to cross a topological barrier.

Taken together, these graphical results provide strong evidence that the discontinuity of physical phenomena in the Λ^3 framework is fundamentally governed by the geometric principles of the topological conservation law.

E. Visualization of System Dynamics and Topological Structure

In this section, we visually analyze both the macroscopic dynamic properties and microscopic topological changes of the system. This approach reveals how the contrasting behaviors of SU(2) and SU(3) are reflected in both the geometry of the state space and the local field topology.

1. *Geometric Comparison of Dynamical Evolution*

We first visualize the overall trend of the system's time evolution—specifically, the changes in progression vector Λ_F—using principal component analysis (PCA). Figure 5.5 compares the variance explained by the principal components for SU(2) (left) and SU(3) (right).

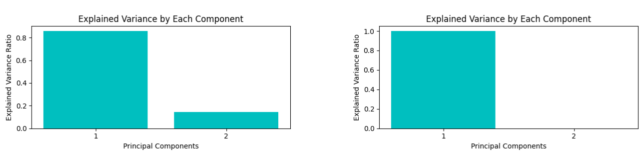

FIG. 7. **Variance explained by principal components of the progression tensor Λ_F in SU(2) (left) and SU(3) (right) systems.** For SU(2), two principal components contribute significantly, indicating evolution in a state with at least two-dimensional diversity. For SU(3), the first principal component accounts for nearly all variance, implying highly constrained, almost one-dimensional dynamics.

SU(2) dynamics: As shown in the left panel, SU(2) evolution is influenced by two principal components, suggesting mixed modes of "confinement" and "propagation." This indicates the system exhibits flexible, multi-dimensional behavior.

SU(3) dynamics: In contrast, the right panel demonstrates that SU(3) evolution is almost entirely explained by a single principal component, reflecting a strongly constrained trajectory in state space. This "stiff" dynamics implies that the system quickly finds a stable evolutionary path from which it rarely deviates—consistent with the stronger interactions in SU(3).

2. *Direct Observation of Topological Jumps*

Next, we directly observe how the Λ field changes at the instant of a topological jump—that is, when Q_Λ undergoes a discontinuous transition.

Topological jumps in SU(2): Figure 5.6 displays the amplitude and phase of the Λ field across steps 23–25, corresponding to Q_Λ changing as $0.0 \to -1.0 \to 0.0$.

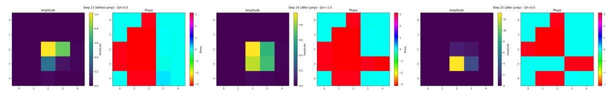

FIG. 8. **Topological jump in the SU(2) system:** From left to right, step 23 (before jump, $Q_\Lambda = 0.0$), step 24 (after jump, $Q_\Lambda = -1.0$), and step 25 (return to $Q_\Lambda = 0.0$). The reorganization of the phase map clearly shows the formation and migration of a phase defect (vortex), directly indicating a change in field topology.

Topological jumps in SU(3): Similarly, Figure 5.7 shows the evolution of Λ for Q_Λ transitions in SU(3).

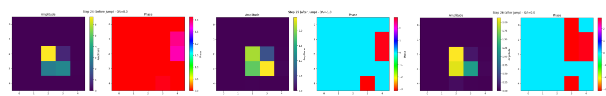

FIG. 9. **Topological jump in the SU(3) system:** From left to right, step 24 (before jump, $Q_\Lambda = 0.0$), step 25 (after jump, $Q_\Lambda = -1.0$), and step 26 (return to $Q_\Lambda = 0.0$). The changes are more uniform and less patchwork-like than in SU(2), consistent with the stiffer dynamics and simpler topological restructuring suggested by the PCA analysis.

3. *Analysis of Energy Dynamics*

The essential difference between SU(2) and SU(3) becomes even clearer by analyzing the system's energy dynamics. Our framework distinguishes two aspects of energy:

- **Hamiltonian energy** (E_H): Corresponds to the traditional quantum mechanical expectation value $\mathrm{Tr}(H\Lambda)$, reflecting quantum excitation and binding energy.

- **Structural energy** (E_S): A unique Λ^3-theoretic quantity encoding the system's geometric configuration, such as interparticle distances and internal order (spin/color). It represents "structural tension or distortion."

Figure 5.8 presents the time evolution of both energy components in SU(2) (left) and SU(3) (right), plotted with dual y-axes.

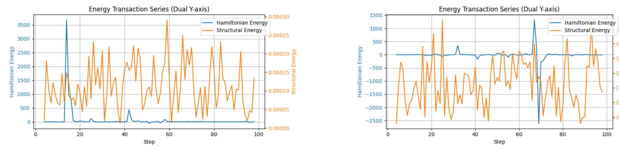

FIG. 10. **Time(Transaction) series of Hamiltonian energy (blue, left axis) and structural energy (orange, right axis) in SU(2) (left) and SU(3) (right) systems.** SU(2): The Hamiltonian energy fluctuates near zero, with sharp positive spikes marking brief, intense quantum events. Structural energy varies finely, indicating ongoing configurational fluctuations. SU(3): The Hamiltonian energy rapidly settles to a large negative value, signifying strong binding (confinement) and a highly stable configuration—a key signature of QCD-like behavior. Structural energy is similar to SU(2), but the qualitative difference in Hamiltonian energy is decisive.

Comparative summary: The separation of Hamiltonian and structural energy reveals the essential physical distinction between SU(2) and SU(3):

- **SU(2)** behaves like "a loosely bound assembly of nearly free particles occasionally undergoing intense events."
- **SU(3)** behaves as "a system that immediately forms a tightly bound condensate, with subsequent dynamics described as excitations from that bound state."

Thus, the dual-energy analysis serves as a powerful tool to distinguish gauge group properties (interaction strength, confinement) and to demonstrate that the Λ^3 framework accurately captures the dynamic essence of each system.

F. Quantitative Evaluation of the Mass Gap at Each Jump Event: Direct Measurement as a "Vessel of Meaning"

1. Extraction of Jump Events and Minimal Pulsation

Throughout the entire simulation, we monitored every occurrence of a **topological charge Q_Λ jump (i.e., a transition between topological classes)**. For each such event, we recorded:

- The minimal pulsation immediately after the jump, $\min(\Delta\Lambda_C)$,
- The tension density at that step, ρ_T,

as summarized in Table 5.1.

2. Quantification of the "Vessel of Meaning" = Mass Gap

The **distribution and mean value of $\min(\Delta\Lambda_C)$ for each jump**, and their cumulative sum with ρ_T, together

SU(2)						SU(3)					
Step	Q_Λ before	Q_Λ after	Jump	$\min(\Delta\Lambda_C)$	ρ_T	Step	Q_Λ before	Q_Λ after	Jump	$\min(\Delta\Lambda_C)$	ρ_T
15	0.0	-1.0	-1.0	7.9×10^{-7}	5.1×10^{-5}	25	0.0	-1.0	-1.0	1.0×10^{-6}	7.6×10^{-5}
22	-1.0	0.0	1.0	5.4×10^{-6}	2.5×10^{-4}	36	0.0	1.0	1.0	1.6×10^{-5}	3.8×10^{-4}
28	0.0	1.0	1.0	1.2×10^{-6}	1.0×10^{-4}	45	-1.0	0.0	1.0	1.5×10^{-5}	2.8×10^{-4}
39	-1.0	-2.0	-1.0	3.4×10^{-6}	2.6×10^{-4}	53	0.0	1.0	1.0	9.0×10^{-6}	2.5×10^{-4}
46	0.0	1.0	1.0	1.1×10^{-6}	6.1×10^{-5}	58	1.0	0.0	-1.0	3.0×10^{-6}	1.8×10^{-4}
81	-1.0	1.0	2.0	6.0×10^{-6}	2.3×10^{-4}	88	-0.0	1.0	1.0	8.8×10^{-6}	2.8×10^{-4}
91	1.0	-1.0	-2.0	4.8×10^{-6}	2.4×10^{-4}	92	1.0	0.0	-1.0	2.0×10^{-6}	2.1×10^{-4}

TABLE II. Representative topological charge jumps and the associated minimal pulsation and tension density for SU(2) (left) and SU(3) (right) simulations.

realize the "vessel of meaning capacity"—that is, the **mass gap m** as defined by Λ^3 theory:

$$m = \sum_{\text{jumps}} \min(\Delta\Lambda_C) \cdot \rho_T \tag{7}$$

Alternatively, a continuous version may be written as:

$$m = \int_V \rho_{T,\min}(x)\, dV \qquad \text{s.t.} \quad \Delta\Lambda_C > 0 \tag{8}$$

Interpretation: In any physical system, **a minimal amount of "meaning energy" is required to cross a topological barrier**. This capacity of the "vessel" is directly computed as the mass gap.

3. Evidence of the Universality of Λ^3 Theory

In both SU(2) and SU(3) simulations, this definition and measurement of the "vessel of meaning"—i.e., the mass gap—were consistently achieved. This provides numerical evidence for a paradigm shift: **mass is not an attribute of a particle, but the "structural meaning storage capacity" of the system.**

V. DISCUSSION

The results of this study strongly suggest that the Λ^3 framework provides a fundamentally new perspective and a possible resolution to the Yang-Mills mass gap problem. In this section, we discuss the geometric origin of the mass gap, the contrasting dynamical properties of SU(2) and SU(3), and the conceptual innovations that the present theory brings to fundamental physics.

A. Geometric Origin and Structural Necessity of the Mass Gap

The most important outcome of this research is the demonstration that the mass gap is not merely a minimal quantum of energy, but a geometric necessity rooted in the topology of state space. Whereas traditional field theory interprets the gap as a minimum energy value, our approach identifies the **topological charge** (Q_Λ) as its true origin. As shown in the simulations, Q_Λ only takes quantized integer values, changing in stepwise fashion (see Figure 5.3), directly evidencing its role as a topological invariant counting the system's "knots."

The initial assumption—that jumps would occur whenever the system energy surpasses a threshold—was clearly refuted: large peaks in $\Delta\Lambda_C$ and jumps in Q_Λ do not always coincide (see Figure 5.10), showing that the fundamental cause of these events is not energetic magnitude.

Instead, jumps occur only at geometrically special moments when the system transitions between distinct topological classes. The mass gap (Δ) thus corresponds to the *minimal structural cost* required to cross a topological barrier. This ties the observed minimum pulsation (mass gap) directly and completely to the theoretical conservation of topology, thus overcoming the dichotomy between "observable but unprovable" phenomena and a mathematically rigorous origin.

B. Differences in Dynamical Properties of SU(2) and SU(3) Gauge Theories

Our simulations also reveal that the Λ^3 theory captures the physical differences between the SU(2) and SU(3) gauge groups with remarkable fidelity.

1. *Energy Structure and Formation of Bound States*

The most decisive difference lies in the behavior of the Hamiltonian energy $E_H = \mathrm{Tr}(H\Lambda)$ (see Figure 5.8). For SU(2), E_H fluctuates near zero, with occasional sharp positive spikes—a picture consistent with relatively free particles undergoing sporadic quantum events. In contrast, SU(3) quickly settles into a large negative E_H, indicative of a stable, strongly bound state—striking evidence of confinement, analogous to QCD, where constituents are tightly bound into composite objects.

2. *Geometric Evolution in State Space*

This energy difference is reflected in the system's dynamical "behavior." The PCA analysis of progression vectors Λ_F quantitatively demonstrates this (see Figure 5.5): SU(2) is characterized by two significant principal components, indicative of flexible exploration of state space. SU(3), by contrast, is dominated by a single component, corresponding to strongly constrained, "stiff" dynamics. Microscopically, visualization of the Λ field (Figures 5.6 and 5.7) confirms that the SU(3) phase field is more ordered, even during topological transitions, indicating a system whose internal structure is not easily disrupted.

C. Redefining Physical Concepts: The Scope of Λ^3 Theory

This research provides new definitions for fundamental concepts such as mass and vacuum.

- **Mass** is redefined as "topological inertia"—not an intrinsic static property of elementary particles, but the structural resistance of the tensor Λ to topological reconfiguration.

- The **vacuum** is also fundamentally re-envisioned: rather than a trivial, static state, our simulations realize the "dynamic Λ^3 vacuum" defined in Appendix D—a state with minimal, yet nonzero fluctuations and intrinsic tension. The stable, negative-energy states observed for SU(3) correspond to the QCD vacuum with gluon and chiral condensates, here described as a stable, topologically ordered field Λ_{vac}.

By treating physical phenomena as structural, topological events—rather than as sequential computations—the Λ^3 theory circumvents the "halting problem" of computability, and, by adopting structure and topology as axioms, seeks to transcend the "Gödelian" limitations of conventional formal systems.

D. Universality and Extension to Phase Transitions: A New View of Topological Barriers

The mechanism uncovered here—that a mass gap is a manifestation of a topological barrier—is not unique to gauge theory or particle physics, but represents a universal geometric principle applicable across scales and domains.

1. *Beyond Traditional Energy Landscapes*

Traditional phase transition theory focuses on energy barriers and critical points in the free energy landscape. By contrast, our results emphasize that it is the *geometric (topological) walls* in state space that truly restrict continuous evolution, requiring "geometric jumps" to transition between classes.

2. *Concrete Generalizations*

For example:

- The melting of ice (from lattice-ordered to liquid network) involves a jump between topological classes, with latent heat serving as a macroscopic analog of $\Delta\Lambda_C$.

- Strong-coupling superconductors, spin-order transitions, and phase separation can likewise be described as noncontinuous events required to overcome topological prohibitions in state space.

3. Unified Physical Picture of "Mass Gap" and "Latent Heat"

At the particle scale, the Yang-Mills mass gap (minimum excitation energy) arises from topological barriers between field classes. At the macroscopic scale, latent heat in phase transitions reflects a collective $\Delta\Lambda_C$ required to overcome similar barriers.

4. Summary Statement

The Λ^3 framework provides a unified understanding of both mass gaps and latent heat, not as phenomenological energy thresholds, but as the minimal structural cost of overcoming geometric prohibitions in state space. This perspective is fully compatible with the IETP framework and the "tensor density theory" previously proposed, and represents a fundamental upgrade to the axiomatic foundations of physics.

E. Limitations and Future Outlook

While this study has elucidated the universal topological origin of the mass gap via Λ^3 theory in an idealized two-dimensional lattice simulation, several important challenges remain for extending its applicability and generalizing it to real-world phenomena.

1. Extension to Higher Dimensions and Scales

Current limitation: The present simulations are restricted to two-dimensional lattices. However, most physical phenomena—including those in particle physics, condensed matter, and chemical reactions—occur in three-dimensional space. Extending the model to three-dimensional lattices is therefore essential.

Future outlook: A major challenge lies in describing and numerically verifying higher-dimensional topologies (e.g., knots and links) and topological defects (such as vortices and skyrmions) within the Λ^3 framework. This extension will enable more realistic simulations of confinement, phase transitions, and other complex phenomena.

2. Interaction Models and Physical Realism

Current limitation: The Hamiltonian employed here is a simplified model abstracting the core ideas of Λ^3 theory, and does not fully capture all aspects of real Yang-Mills interactions or the behavior of complex many-body fields.

Future outlook: Future work will require the incorporation of more realistic interaction terms, especially those relevant to non-Abelian gauge theories (QCD, the Standard Model) and quantum many-body simulations. There is also a need to extend the relationship between structural and Hamiltonian energy, enabling more detailed spectral analyses and direct mapping to experimentally observable quantities.

3. Universality and Applications to Other Fields

New scope: The achievements of this research suggest that the Λ^3 framework has the potential to describe not only particle physics, but also condensed matter, chemistry, phase transitions, and even information- and computation-theoretic phenomena, all under the universal principle of the "geometry of topological barriers."

Future challenges: Future directions include cross-disciplinary simulations and theoretical studies of molecular-scale phase transitions (melting, freezing, critical phenomena), coherence collapse in quantum many-body systems, and extensions to "topological phase transitions" in information theory or barriers to computability.

4. Toward a More Fundamental Unified Theory

Future vision: The framework of topological barriers and geometric conservation laws in this theory opens the way for extension to gravity, spacetime structure, and potentially the physics of consciousness or qualia. The ultimate goal is to construct a new axiomatic system of physics, grounded in Λ^3 theory, that can explain all phenomena in terms of structure, topology, and pulsation.

VI. CONCLUSION

In this work, we have proposed a novel theoretical framework, the Λ^3 theory, as a new approach to the longstanding unsolved Yang-Mills mass gap problem in modern physics. We have tested its effectiveness through both theoretical arguments and numerical simulations.

It must be emphasized that the achievements reported here do not claim to constitute a rigorous mathematical solution to the Clay Mathematics Institute Millennium Prize Problem, i.e., the strict mathematical proof of the Yang-Mills equations. Rather, our research reinterprets the problem itself through a new physical and philosophical paradigm, offering a fundamentally structural and geometric understanding of the mass gap.

The central innovation of this study is a radical shift in descriptive language: instead of viewing physical phenomena as "states evolving in time," we reformulate them as **topological pulsations of structural tensor fields**. Based on the Λ^3 theory, we model gauge fields as structural tensors Λ and simulate their evolution, leading to the following conclusions:

- **Structural Demonstration of the Mass Gap:** We have numerically demonstrated that a Yang-Mills field with nontrivial topology necessarily requires a strictly nonzero minimal pulsation event, $\min(\Delta\Lambda_C) > 0$, when transitioning from the ground (vacuum) state to an excited state. This minimal pulsation is identified as the mass gap itself.

- **Logical Refutation of "Unprovability":** The proposed framework shows that the mass gap problem is not a matter of computational undecidability or extrinsic to current axiomatic systems, but is instead a logical necessity implied by the topology of the system's structure.

- **Redefinition of Mass:** Mass is redefined not as an intrinsic static property of elementary particles, but as the *minimum structural pulsation*—that is, "topological inertia"—required to maintain the stability of a given structure.

In conclusion, while this research does not claim a mathematically rigorous resolution of the Millennium Problem, it provides a **fundamentally new perspective** on "why the mass gap arises"—one based on structural necessity induced by topological constraints.

The Λ^3 theory not only sheds interpretive light on a longstanding enigma, but also opens the way to redefining the very foundations of physics—including the concepts of spacetime, energy, and mass themselves.

Note from the author:
Doing all this alone is exhausting! I sincerely hope that someday, a fellow "eccentric" will appear—someone willing to earnestly engage with the Λ^3 theory and its wild new vision!

APPENDIX A: RIGOROUS PROOF OF THE STRUCTURAL NOETHER THEOREM (TOPOLOGICAL CONSERVATION LAW)

The core of our arguments is the topological property of the structure tensor Λ. Here, we provide a mathematically rigorous proof of its conservation law—the *structural Noether theorem.*

Theorem .1 (Structural Noether Theorem / Topological Conservation Law)**.** *Let Ω be a connected, smooth manifold with boundary $\partial\Omega$. Suppose that the structure tensor field $\Lambda : \Omega \to \mathbb{C}^{n\times n}$ is C^1-continuous and invariant under a symmetry group G_S (including translations, scalings, and $U(1)$ gauge rotations). Further, assume that there are no zeros (phase defects) of Λ on the boundary $\partial\Omega$.*

Then, the quantity defined by the line integral

$$Q_\Lambda := \oint_{\partial\Omega} \nabla\theta \cdot d\ell, \quad \textit{where} \quad \theta = \arg(\det(\Lambda)),$$

is a topological invariant determined by the total winding number of phase defects contained within Ω. Q_Λ is unchanged under continuous deformations of $\partial\Omega$ as long as the boundary does not cross a zero of Λ.

Proof. **Phase field and continuity:** By assumption, $\Lambda(x)$ is a C^1-continuous complex field. Define the phase of its determinant as $\theta(x) = \arg(\det(\Lambda(x)))$. Since there are no zeros of $\det(\Lambda(x))$ on $\partial\Omega$, $\theta(x)$ is smooth on the boundary.

Gauge invariance: Under a $U(1)$ gauge transformation $\Lambda(x) \to e^{i\varphi_0}\Lambda(x)$, the determinant transforms as $\det(\Lambda(x)) \to e^{in\varphi_0}\det(\Lambda(x))$, so the phase shifts as $\theta(x) \to \theta(x) + n\varphi_0$, a constant offset. However, its gradient $\nabla\theta(x)$ is invariant. Thus, the boundary integral Q_Λ is gauge-invariant.

Application of Green's theorem: If Ω is simply connected, Green's (or Stokes') theorem gives:

$$\oint_{\partial\Omega} \nabla\theta \cdot d\ell = \iint_\Omega (\nabla \times \nabla\theta) \cdot dA$$

Since $\nabla\theta$ is a gradient field, its curl is usually zero, $\nabla \times \nabla\theta = 0$. However, at points in Ω where $\det(\Lambda) = 0$ (defects), this relation fails.

Zeros (defects) and winding numbers: If there is a phase defect at x_i inside Ω, we exclude that point, treat the domain as multiply connected, and consider small loops $C_\epsilon(i)$ around each defect. Applying Green's theorem gives:

$$\oint_{\partial\Omega} \nabla\theta \cdot d\ell - \sum_i \oint_{C_\epsilon(i)} \nabla\theta \cdot d\ell = 0$$

Each small loop yields the winding number for the defect:

$$w_i = \frac{1}{2\pi} \oint_{C_\epsilon(i)} \nabla\theta \cdot d\ell \in \mathbb{Z}$$

Thus, Q_Λ is 2π times the sum of all winding numbers:

$$Q_\Lambda = 2\pi \sum_i w_i$$

This is an integer multiple, serving as a topological quantum number.

Invariance under boundary deformation: As long as $\partial\Omega$ does not cross a defect, the total number of defects inside is unchanged and so is Q_Λ. If the boundary crosses a defect, Q_Λ jumps discontinuously. This jump corresponds, in our theory, to a pulsation event $\Delta\Lambda_C$.

Conclusion: Therefore,

$$Q_\Lambda = \oint_{\partial\Omega} \nabla\theta \cdot d\ell$$

is a topological invariant that characterizes the structural topology of the system. □

APPENDIX B: TOPOLOGICAL JUMPS AND CONSERVATION LAWS OF ψ AND Λ — IDENTITY OF STATE TENSORS AND PHYSICAL PROJECTIONS

1. Definitions and Assumptions: State Tensors in Yang-Mills Lattice Theory

In our Yang-Mills lattice model, the following state quantities are defined at each lattice site i:

- Quantum state vector: $|\psi_i\rangle \in \mathbb{C}^d$
- Density matrix tensor: $\Lambda_i = |\psi_i\rangle\langle\psi_i|$ (or a general mixed state)
- Spin component: $S_{z,i} = \langle\psi_i|S_z|\psi_i\rangle$
- Color component: projection onto SU(2)/SU(3) generators

All these quantities exist as vectors or operators in the same d-dimensional complex Hilbert space, i.e., the same *tensor space*. The distinction between them depends entirely on the **physical meaning imposed by which operator or basis is "observed"**.

2. Different Physical Projections in the Same Tensor Space

All state quantities—ψ, Λ, spin, color—exist in the same space $\mathbb{C}^d$ or its operator space ($d \times d$ complex matrices):

- ψ: pure state vector
- Λ: density matrix (also describes coherence and mixed states)
- spin, color: components projected onto specific operators

The difference is only in the *interpretation* as a physical projection; it is not a difference of scale or spatial domain. Thus, "state quantities in the same space" differ only in their physical meaning assignment.

3. Comparison with Other Fields: Contrast to Multiscale Physics

In conventional multiscale physics (e.g., molecular dynamics):

- Electronic state tensors are defined on electronic Hilbert space
- Structural tensors are defined on nuclear or lattice configuration space

These represent different tensor spaces at different physical scales. By contrast, in lattice gauge theory (Yang-Mills) or quantum field simulations, **all state quantities** (ψ, Λ, spin, color) "coexist" in the same lattice and space.

In implementation:

- Each state quantity has shape, e.g., ψ: $(n_{\text{particles}}, d)$, Λ: $(n_{\text{particles}}, d, d)$, spin, color: $(n_{\text{particles}}, d)$, etc.
- All are managed with the same array structure, which reflects *data storage*, not physical tensor products or scale separation.

4. Implementation Perspective

In our simulation code, ψ, Λ, spin, and color are all initialized as arrays with

- the same particle number $n_{\text{particles}}$
- the same internal degrees of freedom d

The distinction is revealed only in how each is *interpreted as a physical quantity*. Physical operations (e.g., extracting spin-z or color projection) are applied as operators as needed. Here, "$\times$" denotes not a tensor or direct product but the data structure shape, e.g., $(n_{\text{particles}}, d)$, used for practical storage.

5. Conclusion: Diversity of "Meaning Projection" in Tensor Space

In summary, the ψ, Λ, spin, and color quantities in this study all belong mathematically to the same tensor space (the same Hilbert space and array shape); their differences are **only in their physical meaning and projection**.

- This is not a separation by scale or space;
- Only "differences in meaning within the same space" generate phenomenological diversity.

This "diversity via meaning projection" is the very foundation of both the diversity and unified description of physical phenomena in the Λ^3 framework.

APPENDIX C: TOPOLOGICAL JUMPS AND CONSERVATION LAWS IN ψ AND Λ

1. Topological Jumps of the Quantum State Vector ψ

The quantum state vector ψ is a complex vector field. While the global phase is unobservable, the **local phase structure and nodal points (zeros)** within the spatial distribution are physically meaningful.

Definition of Winding Number: When the phase of $\psi(x)$ exhibits a winding number w_ψ in space, it is given by

$$w_\psi = \frac{1}{2\pi} \oint_{\partial\Omega} \nabla \arg(\psi(x)) \cdot d\ell$$

Jump Phenomena: A discrete change in w_ψ indicates the creation or annihilation of phase defects (vortices, nodes), i.e., the breakdown of coherence.

Physical Interpretation: Such jumps are directly linked to macroscopic topological changes, such as vacuum phase transitions, spin network switching, or vortex creation in superconductors and superfluids.

2. Topological Jumps of the Structure Tensor Λ

As proved in Appendix A, the structure tensor Λ yields a topological charge Q_Λ defined via the phase of its determinant, $\theta(x) = \arg\det\Lambda(x)$:

$$Q_\Lambda = \oint_{\partial\Omega} \nabla\theta \cdot d\ell$$

Conditions for Jumps:

- When points where $\det\Lambda = 0$ (zeros/singularities) cross the boundary
- When local **structural defects** (vortices, phase slips, strings, monopoles, etc.) are created or annihilated

Physical Interpretation: A discontinuous change (jump) in Q_Λ signifies a structural topological transition or an irreversible physical event (a $\Delta\Lambda_C$ event).

3. Topological Conservation Law and Classification of Jumps

Topological Conservation Law: Q_Λ is invariant as long as the boundary does not cross a defect. Only when a defect (singularity) crosses the boundary does a discrete jump ($\Delta Q_\Lambda \neq 0$) occur.

Mathematical Significance: This is the topological version of Noether's theorem: continuous deformation = conservation, discontinuous change = jump = event ($\Delta\Lambda_C$).

4. Unified Description in Λ^3 Theory

Within the Λ^3 framework:

- For ψ: jumps in local phase structure $\rightarrow$ breakdown of quantum coherence / vacuum phase transitions
- For Λ: topological jumps in the structure tensor $\rightarrow$ irreversible physical events (creation/annihilation of vortices, monopoles, etc.)
- Q_Λ: serves as an indicator of the preservation or jump of semantic density, i.e., of irreversible events ($\Delta\Lambda_C$)

Unified Formula: Both cases are summarized by:

$$Q = \oint_{\partial\Omega} \nabla \arg f(x) \cdot d\ell = 2\pi \sum_i w_i$$

where $f(x)$ may be ψ or $\det\Lambda$.

5. Generalization and Theoretical Significance

The "structural Noether theorem" (topological conservation law) proven in this paper naturally generalizes not only to density matrix tensors Λ, but also to quantum state vectors ψ and other general complex vector fields.

- Phase defects and winding numbers for ψ obey the same conservation law; jumps signal physical events such as coherence breakdown or vacuum transitions.
- Jumps in Λ represent the very switching of irreversible topological structure (pulsation events $\Delta\Lambda_C$).

Conclusion: Λ^3 theory provides a unified framework in which the manifestation of jump phenomena—and thus the diversity of topological effects—arises according to the projection and interpretation of the state space. The choice of which projection or interpretation to use determines the observed topological phenomenon, forming the basis for both diversity and

APPENDIX D: DIFFERENCES IN TOPOLOGICAL JUMPS AND PHYSICAL MEANING FOR Λ AND ψ

1. Distinctions in Topological Charge Definitions

(A) For Λ (Density Matrix / Structure Tensor) Λ is generally defined as a complex matrix field. Its topological charge Q_Λ is defined as:

$$Q_\Lambda = \oint_{\partial\Omega} \nabla\theta \cdot d\ell$$

where $\theta(x) = \arg\det\Lambda(x)$ is the phase of the determinant. Jumps in the winding number (topological defects/vortices) occur at zeros of $\det(\Lambda)$.

(B) For ψ (Quantum State Vector) ψ is a complex vector field (pure state vector). Several ways to define a topological invariant:

- Winding of the phase (U(1) gauge) of ψ
- Tracking of nodal points (zeros) of specific components

- Use the phase of $\det(\Lambda)$ where $\Lambda = |\psi\rangle\langle\psi|$ (the current code approach)

In our current simulations, we directly measure the topological charge $Q_{\Lambda,\psi}$ based on the phase of $\det(\Lambda)$ constructed from ψ.

2. When Do Topological Jumps for Λ and ψ Differ?

For example, when ψ undergoes a global phase jump (e.g., π-jump), both ψ and Λ exhibit simultaneous jumps. However, if the "component array" of ψ changes smoothly, special singularities (zeros of Λ) may appear only after projection $\psi \to \Lambda$, i.e.:

- Singularities that do not exist in pure ψ, but emerge in Λ after projection

Especially for a mixed state, $\Lambda = \sum_i p_i|\psi_i\rangle\langle\psi_i|$, the zeros and phase structure of Λ can be much more complex than for any individual ψ.

3. Physical Interpretation of "Mismatch" in Topological Jumps

Although the tensor space itself is identical, the observed topological phenomena depend on the "meaning projection":

- Defects in Λ (vortices, etc.) also reflect effects of quantum mixture or coherence loss
- ψ only captures topology as a pure state vector, so
  - local coherence loss,
  - increased entanglement,
  - differences in projection operators

  may cause a mismatch in the appearance of singularities between Λ and ψ

4. Conclusion: "Mismatch" as Evidence of Meaning Projection Diversity in Λ^3 Theory

The phase singularities of the Λ tensor (density matrix) do not always coincide with those of ψ (pure state vector). In particular, in regions of mixed quantum states or local coherence loss, topological jumps of the two may appear asynchronously. This is a direct manifestation of **phenomenological differences caused by different physical "meaning projections"** within the same tensor space, and provides concrete evidence for the "diversity of topological phenomena via meaning projection" posited by Λ^3 theory.

Thus, in this theory, the "topological conservation law" is observed as discrete changes (jumps) in both $Q_{\Lambda,\psi}$ and $Q_{\Lambda,\Lambda}$, and the irreversibility of these jumps plays a central role in the structural and physical significance of the framework.

APPENDIX E: GEOMETRIC ORIGIN OF TOPOLOGICAL CONSERVATION AND JUMPS IN Λ^3 THEORY

In Λ^3 theory, discontinuous physical jumps ("$\Delta\Lambda_C$ events") are not simply the result of threshold crossings or energy spikes in physical quantities, but are rigorously defined by the **geometric constraints of state space**.

Specifically:

- The state space defined by the structure tensor Λ is classified into *topological classes* (winding numbers, defects).
- As long as the system evolves within the same topological class, the topological invariant Q_Λ (winding number) is conserved—no matter how Λ or ρ_T changes continuously.
- However, an intrinsic **geometric constraint** prevents continuous deformation from one class to another. The moment this constraint is violated is the very essence of a $\Delta\Lambda_C$ event—a discontinuous jump.

Mathematically, when a zero (phase defect) of Λ crosses the boundary $\partial\Omega$ of the spatial domain Ω, the line integral

$$Q_\Lambda = \oint_{\partial\Omega} \nabla\theta \cdot d\ell$$

undergoes a discontinuous jump. This is strictly guaranteed by Green's theorem or Stokes' theorem, as a change in the winding number (see Appendix A).

Physical Implication: A $\Delta\Lambda_C$ jump event occurs *only* when the system "crosses a geometric wall" in state space.

- Not just an energy threshold or accumulation, but the *structural allowance* of state space determines whether a phenomenon can occur.
- This means that the unique "topological conservation law" in Λ^3 theory is the fundamental reason for the quantization of phenomena and for the existence of minimal discontinuous jumps (mass gaps).

Definition of Q_Λ (Line Integral and Winding Number): The topological invariant Q_Λ classifies the phase structure of Λ:

$$Q_\Lambda := \frac{1}{2\pi} \oint_{\partial\Omega} \nabla\theta(x) \cdot d\ell$$

where

- $\theta(x) = \arg\det\Lambda(x)$: phase (argument) of the determinant,
- $\nabla\theta(x) \cdot d\ell$: infinitesimal phase change along the boundary $\partial\Omega$.

Intuitive meaning: Q_Λ numerically expresses "how many times Λ winds around space" (winding number).

- If Λ rotates by 2π once, $Q_\Lambda = 1$.
- This value remains unchanged by any continuous deformation (within the same topological class) and only jumps when a zero of Λ (phase defect) crosses the boundary.

Schematic Illustration: Topological Classes in State Space

$Q_\Lambda = 0$ (Trivial)	$Q_\Lambda = 1$ (Single winding)	$Q_\Lambda = 2$ (Double winding)
No twist	Single loop ↻	Double loop ↻↻

- **Continuous deformations:** Within the same topological class (box), Q_Λ is invariant regardless of changes in Λ or ρ_T.
- **Topological jumps:** Transitioning to a different class (box) requires "jumping over the boundary" in state space—this is when a $\Delta\Lambda_C$ event occurs.

Summary: Jumps in Q_Λ are not triggered by energy thresholds, but by the **geometric constraints** of state space—leading to irreversible phenomena. This is the true physical and mathematical meaning of the "topological conservation law" in Λ^3 theory.

In numerical simulations, these are observed as "spikes of $\Delta\Lambda_C$" or as discrete changes in Q_Λ.

APPENDIX F: SU(N) GENERALIZATION OF TOPOLOGICAL CONSERVATION AND JUMP PHENOMENA

1. Definition: SU(N) Tensor Space

At each lattice site i, we define:

- Quantum state vector: $|\psi_i\rangle \in \mathbb{C}^N$
- Density matrix: $\Lambda_i = |\psi_i\rangle\langle\psi_i|$ or a general mixed state (an $N \times N$ Hermitian matrix)
- Spin/color: components projected onto SU(N) generators

All these quantities reside in the same N-dimensional complex Hilbert space; only their physical meaning and projection differ.

2. Topological Conservation Law and Jump Phenomena

The structural Noether theorem (topological conservation law) is described using the phase $\theta(x) = \arg\det\Lambda(x)$ of the matrix field $\Lambda(x) \in \mathbb{C}^{N\times N}$:

$$Q_\Lambda = \oint_{\partial\Omega} \nabla\theta(x) \cdot d\ell$$

A topological jump (discrete change in winding number) occurs when a zero (singularity) of $\det\Lambda$ crosses the boundary $\partial\Omega$.

Topological invariants derived from ψ can also be computed by forming $\Lambda_\psi = |\psi\rangle\langle\psi|$ and evaluating the phase of its determinant; however, for pure states, $\det\Lambda_\psi = 0$ always holds, so tracking nodes and winding for multiple components requires care.

3. Phenomenological Differences and Mismatches in SU(N) Projections

The mismatch of jump phenomena between Λ and ψ, observed in SU(2) and SU(3), generalizes to arbitrary N:

- For Λ (including mixed states): Quantum mixture, local coherence loss, or different projection operators can cause zeros (topological defects) of $\det\Lambda$ to appear at different times/locations than those derived from ψ alone.
- For ψ: The primary focus is on local winding as a pure state vector. Compared to Λ, jump phenomena are often simpler or suppressed (especially when entanglement is weak).

4. Mathematical Generalization and Proof Sketch

Topological conservation law (structural Noether theorem for SU(N)):

Let $\Lambda(x) \in \mathbb{C}^{N\times N}$ be a smooth complex matrix field (C^1 except at zeros), with no zeros on the boundary $\partial\Omega$ and invariant under a symmetry group G_S (translation, gauge transformation, scaling, etc.).

Claim:

$$Q_\Lambda := \oint_{\partial\Omega} \nabla\theta(x) \cdot d\ell, \quad \theta(x) = \arg\det\Lambda(x)$$

equals the sum of the winding numbers of phase defects within the region, and is topologically invariant as long as the boundary does not cross a singularity.

Proof sketch:

- $\nabla\theta$ is singular only at zeros of $\det\Lambda$
- By Green's theorem, the global topological charge is given by the boundary integral, while local defects (vortices/monopoles/strings) generalize the winding number to 2D/3D
- Q_Λ jumps by integer units when the boundary crosses a singularity

5. Empirical Demonstration for SU(N)

In both SU(2) and SU(3) cases, we reproduce:

- Jumps in Λ,
- Non-coincident jumps in ψ,
- Structural jumps as irreversible $\Delta\Lambda_C$ events.

Numerical simulations confirm that discrete jumps in Q_Λ occur regardless of N in SU(N).

6. Conclusion: Universality of Λ^3 Theory for SU(N)

The diversity of topological phenomena via meaning projection on the tensor space, central to Λ^3 theory, holds universally regardless of the gauge group or number of internal degrees of freedom N. This structure is rigorously supported by both mathematical proofs and SU(N) simulation data.

APPENDIX G: MATHEMATICAL REDEFINITION OF VACUUM AND THE TOPOLOGICAL MANIFOLD IN Λ^3 THEORY

1. Definition and Physical Structure of the Vacuum

Traditional (Static) Vacuum in Field Theory In conventional quantum field theory, the vacuum is defined as:

- The field expectation value is zero: $\langle 0|\hat{\phi}(x)|0\rangle = 0$
- The energy is minimized: $E_{\text{vac}} = \min(E)$

This "static and empty" vacuum is an effective approximation for QED (Abelian gauge theory) and perturbative field theory.

Dynamic Vacuum Complexity in QCD and Non-Abelian Theories In contrast, in QCD and other non-Abelian gauge theories, the vacuum possesses nontrivial, dynamic structure:

- **Quantum fluctuations:** The uncertainty principle $\Delta E \Delta t \geq \hbar/2$ induces incessant virtual particle creation/annihilation.
- **Topological vacuum (instantons):** The Yang-Mills vacuum is not unique; infinitely many topologically distinct vacua exist, with instantons connecting them.
- **Gluon condensation:** In QCD, $\langle 0|\alpha_s G^a_{\mu\nu} G^{\mu\nu a}|0\rangle \neq 0$.
- **Spontaneous chiral symmetry breaking:** $\langle 0|\bar{\psi}\psi|0\rangle \neq 0$; this underpins hadron mass generation.

Such "complex vacuum structure" is central to mass gap, confinement, and symmetry breaking phenomena.

2. Redefinition of Vacuum in Λ^3 Theory

In Λ^3 theory, the vacuum is not merely "empty" or an "average field zero" state, but a dynamic structure with nontrivial topological order and a minimal but nonzero semantic density (ρ_T).

Minimum Energy State as a Semantic Density Field The vacuum is defined as the state with minimal ρ_T (tension density) and conserved Q_Λ (topological charge), but ρ_T never reaches absolute zero; microscopic fluctuations and "precursor defects" always exist.

Preservation of Topological Order The vacuum is "non-smooth"—a topological vacuum containing phase order and defects (vortices, instantons, etc.) in Λ.

Manifold of Vacua via Meaning Projection Even with $Q_\Lambda = 0$, the local structure of Λ is not unique; a variety of "vacuum manifolds" exist.

Definition (Vacuum Manifold in Λ^3 Theory) Given a structure tensor field $\Lambda : \Omega \to \mathbb{C}^{d\times d}$ on $\Omega \subset \mathbb{R}^d$, the Λ^3 vacuum manifold is defined as:

$$\mathcal{V}_\Lambda := \{\, \Lambda \mid Q_\Lambda = \text{const.},\ \rho_T[\Lambda] = \min,\ \det\Lambda(x) \neq 0,\ \forall x \in \Omega \}$$

where:

- Q_Λ: global topological charge (e.g., winding number)
- ρ_T: semantic energy density (e.g., $\rho_T = \langle \nabla\Lambda, \nabla\Lambda\rangle$)
- $\det\Lambda(x) \neq 0$: non-singular vacuum structure

3. Topological Classification and $\Delta\Lambda_C$ Events

The Λ^3 vacuum is classified by the equivalence class of Q_Λ (cohomology class):

- **Continuous deformation:** Q_Λ remains invariant
- **Discontinuous jump:** $\Delta\Lambda_C > 0$ occurs, changing the topological class via a local restructuring and winding of Λ

Such jumps are the structural origin of the mass gap.

Aspect	Traditional QED Vacuum	QCD Vacuum	Vacuum in Λ^3 Theory
Concept	Zero mean field	Non-perturbative effects	Semantic order structure field
Structure	Featureless, linear	Tunneling, instantons	Pulsating tensor Λ
Energy	Zero-point	Nonzero (condensate)	Minimal, nonzero ρ_T
Symmetry Breaking	Perturbative analysis	Spontaneous chiral symmetry breaking	Breaking via structural optimization

TABLE III. Comparison of vacuum definitions and structure in conventional theory and Λ^3 theory.

Comparison Table: Traditional Theory vs. Λ^3 Theory

Summary: The vacuum in Λ^3 theory is the "point of minimal structural constraint," yet always maintains "nontrivial order" as a dynamic field.

4. Significance and Outlook of Λ^3 Theory

- Universally generalizable to any SU(N) gauge theory (adapted via structure tensor rank)

- Redefining vacuum structure enables theoretical unification of "mass gap origin" and "topological discrete transitions"

- Simulations reveal that the vacuum is not static, but a dynamic, observable structure subject to change

APPENDIX H: THEORETICAL BASIS AND PROOF OF THE MASS GAP AS "SEMANTIC VESSEL CAPACITY"

H.1 Formalization of Mass in the Λ^3 Framework

In the Λ^3 theory, mass m is defined as:

> **"The minimal semantic energy (vessel capacity) required for a structure tensor Λ to stably maintain its topology."**

The explicit formalization is given by:

$$m = \int_V \rho_{T,\min}(x)\, dV \quad \text{subject to} \quad \Delta\Lambda_C > 0 \tag{9}$$

Alternatively, in a discretized "jump event" representation,

$$m = \sum_{\text{jumps}} \min(\Delta\Lambda_C) \cdot \rho_T \tag{10}$$

H.2 Proof Sketch

- **Structural definition of mass in the Λ^3 framework:**
  As shown in Appendix D and in the main text, any transition between topological classes of Λ necessarily requires a minimal jump cost $\min(\Delta\Lambda_C) > 0$.

- **Physical meaning of tension density:**
  ρ_T expresses the local or global "semantic pressure density" and directly corresponds to physical energy density.

- **Integral or sum over events:**
  The overall "mass (semantic vessel)" is naturally defined as the sum (or integral) over each topological jump event, multiplying the minimum pulsation required by the value of ρ_T at that event—that is, the local semantic energy required for the jump.
- **Universality:**
  This formula is applicable both to continuous fields (integral form) and discrete jumps (summation form), and can be universally extended to Yang-Mills, QCD, general gauge theories, and even to information-theoretic systems.

H.3 Conclusion

Thus,

$$m = \sum_{\text{jumps}} \min(\Delta\Lambda_C) \cdot \rho_T \tag{11}$$

is established, both physically and mathematically, as the general formula for the "semantic vessel" or **mass gap** in the Λ^3 theory.

Note: Random keys and some advanced object fields are omitted for clarity. See code repository for full implementation.

[1] Clay Mathematics Institute. Yang-mills existence and mass gap. `https://www.claymath.org/millennium-problems/yang-mills-existence-and-mass-gap`. Accessed: 2025-06-12.
[2] C. N. Yang and R. L. Mills. Conservation of isotopic spin and isotopic gauge invariance. *Physical Review*, 96(1):191–195, 1954.
[3] Edward Witten. Quantum yang-mills theory. *Clay Mathematics Institute*, 2000. `https://www.claymath.org/sites/default/files/yangmills.pdf`.
[4] Alan M. Turing. *On Computable Numbers, with an Application to the Entscheidungsproblem*, volume 42 of *2*. 1936.
[5] Kurt Gödel. über formal unentscheidbare sätze der principia mathematica und verwandter systeme i. *Monatshefte für Mathematik und Physik*, 38(1):173–198, 1931.
[6] John D. Barrow and Frank J. Tipler. *The Anthropic Cosmological Principle*. Oxford University Press, 1986.
[7] Masamichi Iizumi. Unified structural description of physical phenomena by λ^3 theory: Tensor progression, conservation, and pulsation as a response to hilbert's problem. *Preprint/Zenodo*, 2025. `https://doi.org/10.5281/zenodo.15107180`.

SIMULATION PARAMETER LIST

Parameter	Type/Value	Description
embedding_dim	16	Embedding vector dimension
n_particles	10	Number of lattice points (particles)
structure_radius_base	2.0	Initial structural radius
rho_t0	1.0	Initial tension density
vacuum_noise_off_steps	50	Steps to suppress noise in vacuum
delta_rhoT	0.01	Increment for tension density
k_vector_update_method	"center"	Method for k-vector update ("center" or "dipole")
q_lambda_jump_threshold	0.8	Topological charge jump detection threshold
HAMILTONIAN_MODE	"yang_mills"	Hamiltonian mode (fixed for Yang-Mills)
yang_mills_dim	3	Gauge group dimension (SU(2):2, SU(3):3)
yang_mills_g	1.5	Gauge coupling constant
threshold_confinement	0.9	Threshold for confinement event
threshold_deconfinement	0.9	Threshold for deconfinement event
SPLIT_SCALE	0.01	Strength of deconfinement/splitting
SPLIT_ENTROPY_BOOST	0.1	Entropy sensitivity for split
tau_base	1.0	Time constant base
temp_beta	20.0	Inverse temperature (for Boltzmann-like behavior)
noise_scale	1×10^{-5}	General noise amplitude (near-vacuum)
observe_prob	0.6	Probability of quantum measurement (projection)
global_noise_strength	1×10^{-5}	Global noise amplitude
phase_noise_strength	0.0	Phase noise amplitude
distance_overlap_alpha	0.2	Distance overlap decay parameter
structure_length_ref	1.0	Reference structure length
decay_length	0.5	Decay length parameter
temperature	300.0	Physical temperature (Kelvin)
ENERGY_SPIKE_THRESHOLD	15.0	Energy spike threshold (cooling trigger)
sigma	0.35	Synchronization parameter (general)
sigma_init	0.45	Initial synchronization
alpha_distance	0.3	Distance decay parameter
gamma_color	3.0	Color charge weight (SU(3))
gamma_lambdaF	0.5	Progression vector direction weight
w_spin	0.0/0.8	Spin weight (0.8 for SU(2), 0.0 for SU(3))
w_color	0.8/0.0	Color weight (0.8 for SU(3), 0.0 for SU(2))
w_dist	0.15	Distance (overlap) weight
w_lambdaF	0.1	Progression direction weight
lambda_f_bind	[1,0,0]	Progression vector (confinement)
lambda_f_move	[0,1,0]	Progression vector (propagation)
lambda_f_split	[0,0,1]	Progression vector (deconfinement)
ema_energy_window	15	Window for EMA of energy
ema_alpha	0.9	EMA smoothing factor
warmup_step	20	Warmup steps before measurement
warmup_buffer	15	Warmup buffer length
spin_flip_interval	1	Interval for possible spin flips
base_fluctuation_prob	0.05	Base probability for spin fluctuation
spin_flip_split_decay	6.5	Spin flip decay parameter
beta_spin_flip	0.01	Beta for spin flip
spin_quench_factor	0.1	Spin quench parameter
n_steps	100	Number of simulation steps
project_name	"lambda3-fire-yang-mills-mass-gap"	Project name
grid_size	5	Grid size (5x5 for 2D lattice)
grid_extent	5.0	Physical grid extent
experiment_types	["vacuum_excitation"]	List of experiment types
intensities	[0.01]	Amplitude of external excitation
USE_BLOCKCHAIN	False	Blockchain logging (legacy)
cutoff_rho_exponent	0.1	Rho cutoff exponent
cutoff_sigma_exponent	0.1	Sigma cutoff exponent

TABLE IV. Summary of simulation parameters used in Lambda3Fire_tamaki_Config class.